Eötvös-HPS 99-24

On the Einstein-Fine solution of the EPR-Bell paradox*

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The aim of this paper is to make an introduction to the Einstein-Fine interpretation of quantum mechanics and to show how it can solve the EPR-Bell problem. Analyzing the EPR experiment we will find a principal logical loophole in the real spin correlation experiments. The Einstein-Fine interpretation claims that the detection inefficiency we encounter in the experiments is not the effect of the random errors in the analyzer + detector equipment, but it is the manifestation of a pre-settled (hidden) property of the particles. In the second part of the paper I prove the existence of reasonable prism models of the $n \times n$ spin correlation experiment, in which the efficiencies are 50%, independently of n.

CONTENT

* Supported by the OTKA Foundation, No T 015606 .

Introduction

The aim of this paper is to make an introduction to the Einstein-Fine interpretation of quantum mechanics and to show how it can solve the EPR-Bell problem.

To place the Einstein-Fine interpretation in the context, in the first sections I give a short review on the standard interpretations of quantum mechanics and the typical difficulties they suffer from. The EPR-Bell paradox is one of the main problems, which is common for all of these interpretations. Analyzing the EPR experiment we will find a principal logical loophole in the real spin correlation experiments. And this is the point where the Einstein-Fine interpretation appears on the scene. It claims that the detection inefficiency we encounter in the experiments is not (or not merely) the effect of random errors in the analyzer + detector equipment, but it is the manifestation of a pre-settled (hidden) property of the particles.

Following this line of thought, I shall present Fine's simple prism model for the EPR experiment. On this example I shall show how the Einstein-Fine model can be tested experimentally, and we shall compare it with the recent experimental results.

In the first part of the paper I will draw an optimistic picture about the Einstein-Fine interpretation. In final analysis, as we shall see, this optimism is completely justified. There is, however, a serious objection against Fine's concrete prism models. Namely, in a generalized prism model of the $n\times n$ spin correlation experiment the efficiencies tend to zero if $n\to\infty$. In the second part of the paper I shall show that this objection is essentially based on an unjustified symmetry condition. Finally I shall prove the existence of reasonable prism models of the $n\times n$ spin correlation experiment, in which the efficiencies are 50%, independently of n.

The standard interpretations of quantum states

1. According to the two usual interpretations of *quantum state*, we distinguish two major branches of interpretations of quantum mechanics:

(A) Interpretations which assert that *a pure state ψ provides a complete and exhaustive description of an individual system.* A dynamical variable represented by the operator $\hat{X}$ has value x if and only if $\hat{X}\psi = x\psi$.

(B) The *statistical interpretations*, according to which a pure state (and hence also a general state) provides a description of certain statistical properties of an abstract *ensemble* of similarly prepared systems, but need not provide a complete description of an individual system.

These two interpretations yield two different branches of problems. The major difficulty for interpretation (A) is the measurement problem: a measurement process, in general, yields to a coherent superposition of pure states corresponding to macroscopically distinguishable configurations of the measuring apparatus. The existence of such a state raises serious contradictions for interpretation (A): the pointer of the apparatus, for instance, must not have a position at all. And this claim is prima facie conflicting with the definiteness of macroscopically distinct configurations of the measuring apparatus, which we commonly experience and appeal to in the very laboratory practice of testing quantum mechanics itself.

The issue of quantum measurement is no problem for B-theorists. According to the statistical interpretation, such a coherent superposition does not mean that the apparatus "has no definite pointer position” at the end of an *individual* measurement process. For statistical interpretation assigns a state to an *abstract ensemble* of similarly prepared systems. At the end of an individual run of the measurement the pointer may have a definite position at “ x ”, even if the statistical features of the whole ensemble are widely different from those characterized by the eigenstate ψ_x .

2. Since statistical interpretation does not account for the outcome of an individual measurement process, it is a prima facie incomplete description of the world, open for a hidden variable theory.

There are, however, difficulties when we consider the statistical interpretation in details. In order to see these difficulties we will distinguish two further versions of statistical interpretation of the expression $tr(WP_E)$. The first one is the property or quantum event interpretation:

(B1) There exists a *property* or “quantum” *event* $\tilde{E}$ in reality, which occurs with relative frequency $p(\tilde{E}) = tr(WP_E)$. (W is the state operator of the system, P_E is the projector assigned to the measurement outcome in question.)

In this interpretation a measurement outcome E just reveals a corresponding property $\tilde{E}$ of the system. Or at least there exists a “quantum event” $\tilde{E}$, a category of the state of affairs, which is the case when the measurement outcome E occurs. In any case, there exists something, which occurs with relative frequency equal to $tr(WP_E)$. I have claimed in an earlier paper (Szabó, forthcoming) that the property/quantum-event interpretation is *untenable*, because of the violation of the Bell-type inequalities. The obstacle to such an interpretation is that there are no such things the relative frequencies of which violate Bell-type inequalities.

3. Consequently, one might think to reduce statistical interpretation to the so-called *minimal interpretation*:

(B2) $tr(WP_E) = p(E \mid e)$, which means the *conditional* probability of the outcome-event E, *given* that the measurement-preparation e has happened.

In accordance with the everyday laboratory practice, $tr(WP_E)$ no doubt must be equal to $p(E \mid e)$! The controversial question is, of course, whether it means something more. This “something more” must be (and will be, as we shall see hereafter) different from (B1). The reason why we are unsatisfied with the minimal interpretation is, contrary to the other interpretations that are self-contradictory, a metaphysical one: (B2) does not account for *intrinsic properties* of the system revealed by the experiments and described, if only statistically, by the theory.

The EPR-Bell problem

4. The violation of locality is a problem for all above-mentioned interpretations of quantum mechanics. Since the other interpretations are contradictory in themselves, I want to investigate the locality problem in the context of (B2).

The typical example for the violation of locality is the EPR experiment (Fig. 1): We consider the four ‘spin-up’

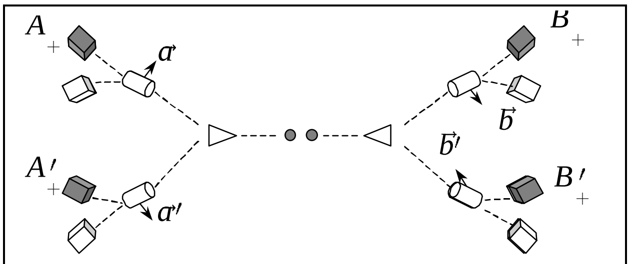

Figure 1 *The EPR experiment with spin-$\frac{1}{2}$ particles*

events in the spin-component measurements in directions $\mathbf{a}, \mathbf{a}'$ and $\mathbf{b}, \mathbf{b}'$. There are random switches (independent agents, if you want) choosing between the different possible measurements on both sides. Let $p(a), p(a')$ and $p(b), p(b')$ be *arbitrary* probabilities with which the different measurements are chosen. We observe the following events in the experiment:

A_+, A'_+:	"the spin of the *left* particle is *up* in direction $\vec{a}, \vec{a}'$" detector fires
B_+, B'_+:	"the spin of the *right* particle is *up* in direction $\vec{b}, \vec{b}'$" detector fires
a, a':	the *left* switch chooses the direction $\vec{a}, \vec{a}'$
b, b':	the *right* switch chooses the direction $\vec{b}, \vec{b}'$

Let $\mathbf{a}, \mathbf{a}', \mathbf{b}, \mathbf{b}'$ be coplanar vectors with $\sphericalangle(\mathbf{a}, \mathbf{a}') = \sphericalangle(\mathbf{a}', \mathbf{b}') = \sphericalangle(\mathbf{a}, \mathbf{b}') = 120^\circ$, and $\sphericalangle(\mathbf{a}', \mathbf{b}) = 0$. We observe the following relative frequencies in the experiment:

$$\begin{aligned} p(A_+) &= \frac{1}{2} p(a) & p(B_+) &= \frac{1}{2} p(b) \\ p(A'_+) &= \frac{1}{2} p(a') & p(B'_+) &= \frac{1}{2} p(b') \\ p(A_+ B_+) &= \frac{3}{8} p(a) p(b) & p(A'_+ B_+) &= 0 \\ p(A_+ B'_+) &= \frac{3}{8} p(a) p(b') & p(A'_+ B'_+) &= \frac{3}{8} p(a') p(b') \end{aligned} \tag{1}$$

5. I need not here enter on the detailed and subtle discussion of concepts like "locality", "factorizeability", "parameter independence", "screening off", "common cause", etc. For, in my view, the original EPR-Bell problem consists in the following simple question of the physicist: Can the EPR experiment be accommodated in a *relativistic* and *deterministic* universe?

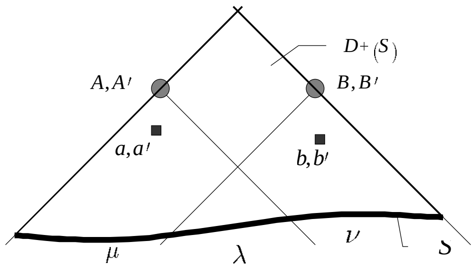

Figure 2 *The space-time diagram of the EPR scenario. All events in the future domain of dependence $D^+(S)$ are predetermined by the Cauchy data along the hypersurface S.*

Figure 2 shows the space-time diagram of one single run of the EPR experiment. In a relativistic and deterministic universe, the Cauchy data along a hypersurface S predetermine everything going on in the future domain of dependence $D^+(S)$. In particular, all future EPR events in the domain of dependence $D^+(S)$ are predetermined by the (partly "hidden") Cauchy data $\{\mu, \lambda, \nu\}$ defined on the three spatially separated regions of hypersurface S. To express this determination we assign a function to each event X in $D^+(S)$:

$$u_X(\mu, \lambda, \nu) = \begin{cases} 1 & \text{if } X \text{ is the case} \\ 0 & \text{otherwise} \end{cases}$$

Because of the spatial separation, the occurrence of event A is independent of the value of parameter ν. Taking into account the similar separations, the following conditions must be satisfied:

$$
\begin{array}{ll}
u_A(\mu,\lambda,\nu) = u_A(\mu,\lambda) & u_B(\mu,\lambda,\nu) = u_B(\nu,\lambda) \\
u_{A'}(\mu,\lambda,\nu) = u_{A'}(\mu,\lambda) & u_{B'}(\mu,\lambda,\nu) = u_{B'}(\nu,\lambda) \\
u_a(\mu,\lambda,\nu) = u_a(\mu) & u_b(\mu,\lambda,\nu) = u_b(\nu) \\
u_{a'}(\mu,\lambda,\nu) = u_{a'}(\mu) & u_{b'}(\mu,\lambda,\nu) = u_{b'}(\nu)
\end{array} \tag{2}
$$

The probabilistic description

6. Assume now that the stochastic feature of the quantum mechanical description of the EPR situation is of epistemic origin, related with the lack of knowledge about all details of the Cauchy data on hypersurface S. What concerns us here is whether this assumption is tenable or not. The statistical ensemble consists of the similar patterns corresponding to the subsequent repetitions of the experiment (Figure 3). As a consequence of the spatial separation, we assume that

$$
p(\mu \wedge \lambda \wedge \nu) = p(\mu)\, p(\lambda)\, p(\nu) \tag{3}
$$

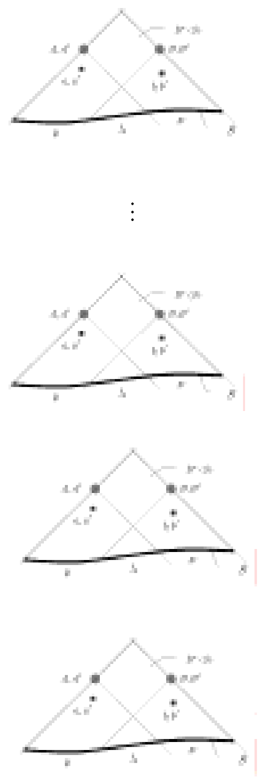

Figure 3. *The statistical ensemble consists of the repetitions of the experiment.*

Now,

$$
\begin{aligned}
p(A) &= \sum_{\mu,\lambda} u_A(\mu,\lambda)\, p(\mu)\, p(\lambda) \\
p(B) &= \sum_{\lambda,\nu} u_B(\lambda,\nu)\, p(\lambda)\, p(\nu) \\
p(a) &= \sum_{\mu} u_a(\mu)\, p(\mu) \\
p(b) &= \sum_{\nu} u_b(\nu)\, p(\nu) \\
p(A \wedge B) &= \sum_{\mu,\lambda,\nu} u_A(\mu,\lambda) u_B(\lambda,\nu)\, p(\mu)\, p(\lambda) p(\nu) \\
p(a \wedge b) &= \sum_{\mu,\nu} u_a(\mu) u_b(\nu)\, p(\mu)\, p(\nu) = p(a)\, p(b)
\end{aligned} \tag{4}
$$

and

$$
\begin{aligned}
p(A \wedge B \,|\, a \wedge b \wedge \lambda) &= \frac{\sum_{\mu,\nu} u_A(\mu,\lambda) u_B(\lambda,\nu)\, p(\mu) p(\nu)}{\sum_{\mu,\nu} u_a(\mu) u_b(\nu)\, p(\mu)\, p(\nu)} \\
&= \frac{\sum_{\mu} u_A(\mu,\lambda)\, p(\mu) \sum_{\nu} u_B(\lambda,\nu)\, p(\nu)}{\sum_{\mu} u_a(\mu)\, p(\mu) \sum_{\nu} u_b(\nu)\, p(\nu)} \\
&= p(A \,|\, a \wedge \lambda)\, p(B \,|\, b \wedge \lambda)
\end{aligned} \tag{5}
$$

So, λ is a stochastic hidden parameter satisfying the "screening off" condition, which from, together with (4), the well known Clauser-Horne inequalities derive immediately:

$$
-1 \leq \underbrace{\begin{array}{r} p(A \wedge B \,|\, a \wedge b) - p(A \wedge B' \,|\, a \wedge b') + p(A' \wedge B \,|\, a' \wedge b) \\ + p(A' \wedge B' \,|\, a' \wedge b') - p(A' \,|\, a') - p(B \,|\, b) \end{array}}_{CH} \leq 0
$$

However, the Clauser-Horne inequalities are violated in case of probabilities (1). ($CH = \frac{1}{8}$) Consequently, as the standard conclusion says, *the EPR experiment cannot be accommodated in a relativistic and deterministic universe.*

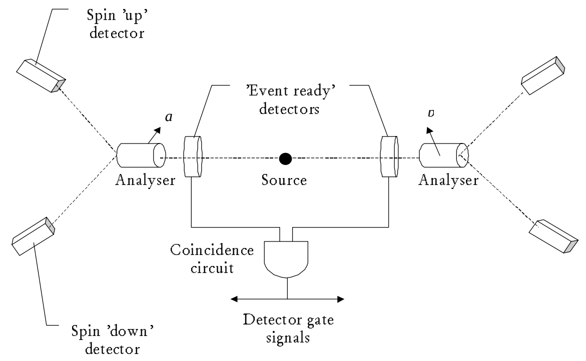

Figure 4 *Apparatus configuration used for Bell's 1971 proof. 'Event-ready' detectors signal both arms that a pair of particles has been emitted.*

Notice that the above derivation of the Clauser-Horne inequalities rests essentially on the assumption (3). Because we did not *assume* the screening off condition (5), but *derived* it from the assumption that the whole scenario is accommodated in a relativistic and deterministic universe, in conjunction with (3). This background of the 'screening off' condition does not appear explicitly in the other derivations of Bell-type inequalities, since the 'screening off' property is taken as an independent assumption. The 'screening off' is, however, either a *fact* derived from *determinism + relativity + (3)*, or, as Cartwright (1987) rightly pointed out, an *assumption* which lacks any foundation.

Loophole in the real EPR experiments

7. There were two well-known loopholes in the earlier EPR experiments: the inefficiency of detection and the imperfect spatial separation of the two wings of the measurement. Both problems have been practically solved in the new experiments performed in the last years. The loophole I would like to concern in this section is a third one, although it is closely related with the detection inefficiency problem. It is worth comparing the original apparatus configuration used for Bell's 1971 proof with the one used in the real Aspect experiment (Fig. 4 and 5).

The original configuration contains two 'Event-ready' detectors, which signal both arms that a pair of particles has been emitted. So, the statistics are taken on the ensemble of particle pairs emitted by the source (or at least, on an ensemble pre-selected by the independent 'Event-ready' detectors).
In the real experiments, however, instead of the event-ready detectors, a four-coincidence circuit detects the 'emitted particle-pairs'. However, this method yields to a *selected* statistical ensemble: only those pairs are taken into account, which coincidentally fire one of the left and one of the right detectors. If the selection went on *completely randomly* then for all X, $p_{selected}(X) = p(X)$, and the above derivation of the Clauser-Horne inequalities would still be valid, and the EPR experiment would still contradict to relativistic determinism.

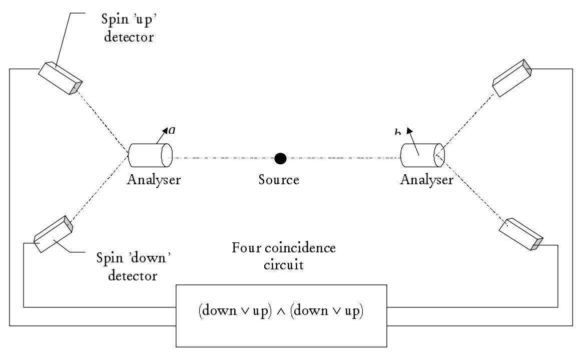

Figure 5 *In the real experiments, instead of the event-ready detectors, a four-coincidence circuit detects the 'emitted particle-pairs'.*

8. However, the Cauchy data $\{\mu, \lambda, \nu\}$ predetermine the whole future behavior of the particle pair (in $D^+(S)$), in particular, they predetermine that the particles can or cannot go through the analyzers. And, *it is quite plausible to assume that the shared part of these data, λ, influences both particles' behavior in the analyzers.* If this is the case, then the actually observed ensemble *is not randomly selected: it depends on the properties of an element of the ensemble, whether it is selected or not.* In other words, not all combinations of μ, ν and λ are selected with equal probability. Therefore, in general,

$$p_{selected}(\mu \wedge \nu \wedge \lambda) \neq p_{selected}(\mu \wedge \nu)\, p_{selected}(\lambda)$$

Consequently, *relativistic determinism does not imply the Clauser-Horne inequalities.*

Thus, the widespread conclusion that the violation of Bell-type inequalities implies the incompatibility between the EPR experiments and relativistic determinism can be mistaken. It is to be emphasized that this claim is based on a *logical* loophole in the real EPR experiments, and not a technical one. It is related with the logical schema of the experiment, independently of the detectors' inefficiency problem; in the above consideration the detector efficiency was taken 100%.

The Einstein-Fine interpretation of quantum mechanics

9. The first question we must raise is whether it is possible to realize an 'event-ready' detection and to perform the measurement on the unselected ensemble. It seems, however, no way to solve this problem: *in practice all conceivable 'event-ready' detectors depolarize or destroy the particles.* Moreover, it seems very difficult to argue against that *this is a uniform feature of all quantum measurements*.

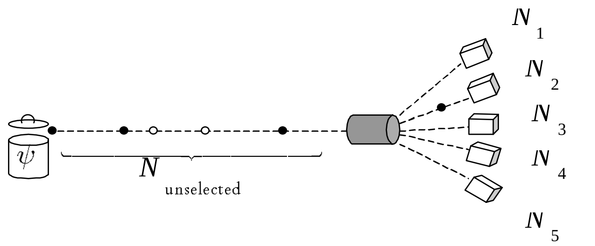

***Figure 6** We have no information about the content of the original unselected ensemble of objects. It is always assumed that each object on which the measurement is performed produces one of the possible measurement outcomes.*

Another question one can ask is the following: Admitting that in the EPR experiments there is such a mechanism, leading to a non-random selection, how is it possible that the laws of quantum mechanics fit so precisely to the experimentally observed probabilities on the selected ensemble? In other words, what does an interpretation of quantum mechanics look like, which naturally accounts for such a biased selection process in the EPR experiment?

To answer this questions consider a typical configuration of a quantum measurement (Fig. 6). We have no information about the content of the original unselected ensemble of objects. It is always assumed that the total number of objects is $N = \sum_i N_i$, where N_i denotes the number how many times the i-th outcome occurred. The theoretical "probabilities" predicted by quantum mechanics are compared with the experimental results in the sense of $tr(WP_i) = \frac{N_i}{N}$, where P_i denotes the projector belonging to the i-th outcome. That is, quantum mechanical "probabilities" are equal to the relative frequencies taken on a *selected* ensemble, namely, on the ensemble of objects producing any outcome (passing the analyzer at all).

In order to understand why are the measured conditional probabilities systematically equal to quantum probabilities, consider a simple case of a measurement a testing the value of a two-

valued ($\{A_+, A_-\}$) observable A (Fig. 7.). Let n^A denote the number of elements, which are predetermined (by the hidden properties) to produce an outcome at all. n_+^A is the number of those elements, which are predetermined to produce outcome A_+. Subset a contains the randomly chosen N elements on which the measurement is performed. Among the measured objects, N_+^A is the number of those which produce outcome A_+. Now, because of the random choice of the measured elements, the conditional probability, for instance, of the outcome A_+, given that the measurement a has been performed, $p(A_+ \mid a)$, must be equal to the relative frequency of elements having property A_+ among those which are capable to produce any outcome of such a measurement, $\frac{n_+^A}{n^A}$.
According to the interpretation we are exposing here this relative frequency is equal to the quantum probability:

$$p(A_+ \mid a) = \frac{N_+^A}{N} = \frac{n_+^A}{n^A} = tr\left(WP_{A_+}\right)$$

In case of two simultaneous measurements (Fig. 8) a similar consideration yields to

$$p(A_+ \wedge B_+ \mid a \wedge b) = \frac{N_{++}^{AB}}{N} = \frac{n_{++}^{AB}}{n^{AB}} = tr\left(WP_{A_+ \wedge B_+}\right)$$

10. The above exposed interpretation of quantum mechanics is nothing else but Arthur Fine's (1982) "Prisms Model", which is, as he claimed (see Fine (1986), p. 52.), nothing but Einstein's statistical interpretation of quantum states.

Without here entering on the details of the historic question whether Einstein's brief and implicit remarks in question should really be regarded as the antecedents of Fine's full-fledged local and realistic interpretation of quantum mechanics, I would like to pose another "historic" question: How is it possible that Fine's interpretation hasn't so far met with a warm response? Interestingly enough, in his later writings Fine himself has interpreted the EPR-Bell problem as if the Einstein-Fine interpretation wasn't exist. "The quantum theory contains just such correlations, … that defy direct causal or common causal explanations" he writes in his (1993, p. 567.), or typically, in his exhaustive analysis of the EPR-Bell problem (Fine 1989) the prism model is not even mentioned.

The reason why Fine's prism model has been ignored is that it has been far too embedded in the context of the hotly disputed *efficiency problem*. Fine, too, frequently appealed to the low effi-

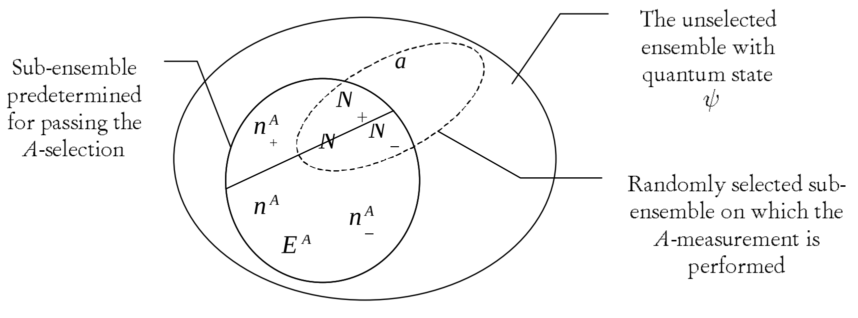

Figure 7 *Subset a consists of elements of the ensemble on which the value of a two-valued observable A has been measured. Let n^A denote the number of elements E^A, which are predetermined to produce an outcome at all. n_+^A is the number of those elements, which are predetermined to produce outcome A_+. Subset a contains the randomly chosen N elements on which the measurement is performed. Among the measured objects, N_+^A is the number of those which produce outcome A_+.*

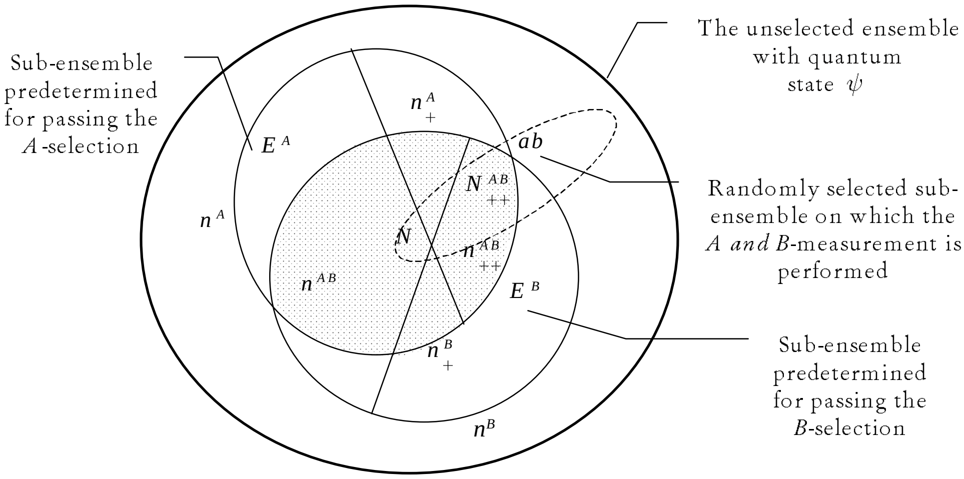

Figure 8 *Subset ab consists of elements of the ensemble on which the value of a two-valued observables A and B have been measured. Let n^A denote the number of elements in E^A, which are predetermined to produce an outcome at all.. n^A_+ is the number of those elements, which are predetermined to produce outcome A_+. We use the same notation for observable B. Subset ab contains the randomly chosen N elements on which the two simultaneous measurements are performed. Among the measured objects, N^{AB}_{++} is the number of those which produce outcome A_+ and B_+*

ciency of the detectors in the actual experiments (see Fine 1989). His readers, however, did not care with his delicate distinction between ‘conspiracy and “conspiracy”’:

> … the efficiency problem ought not to be dismissed as merely one of biased statistics and conspiracies, for the issue it raises is fundamental. Can a hidden variable theory of the very type being tested explain the statistical distributions, inefficiencies and all, actually found in the experiments? If so then we would have a model (or theory) of the experiment that explains why the samples counted yield the particular statistics that they do. (1989, p. 465)

Distinction must be made between the low detection/emission efficiency caused by the random errors in the “analyzer + detector” equipment and the low efficiency that is a systematic manifestation of certain (hidden) properties of the emitted particles. So, those who agree with Bell that

> … it is hard for me to believe that quantum mechanics works so nicely for inefficient practical set-ups and is yet going to fail badly when sufficient refinements are made. (Bell 1987, p. 154.)

are missing the point. Since an adherent to the Einstein-Fine interpretation does not expect that quantum mechanics will fail badly when sufficient refinements are made, but rather that there are principal limits in achieving such refinements. It will be discussed in the next sections what kind of limits we can encounter, and what kind of refinements are achieved in the recent experiments.

A concrete Einstein-Fine-model reproducing the EPR statistics

11. Taking use of Fine's "maximal prism model" (Fine 1982) I would like to present here a simple Einstein-Fine theory reproducing the statistics (1). It is a local, realistic and deterministic hidden parameter model. Local and deterministic in the sense that it does not contradict to a relativistic and deterministic picture we considered in point **6** and realistic, because it admits pre-settled intrinsic properties of the particles, prior to, independent of and revealed by the measurements. Figure 9 shows a parameter space $\Lambda \ni \lambda$ consisting of disjoint blocks of measure $\frac{3}{32}$ and $\frac{1}{32}$ respectively. A point of Λ (a value of the parameter λ) predetermines all events in question. Therefore all events we consider can be represented as subsets of Λ. For instance, assume that $\lambda = \lambda_{\text{example}}$. Then, an a-measurement on the left particle produces neither event "up" nor event "down", while if an a'-measurement is performed then the outcome is "down". At the right wing, if we perform a b-measurement then the outcome is "up", and if the b'-measurement is performed, the outcome is "down". Consequently, in case, for example, we perform an a-measurement on the left particle and a b-measurement on the right one, then there is no coincidence registered, and the particle pair in question does not appear in the statistics of the measurement. On the contrary, if we perform an a'-measurement on the left particle and a b-measurement on the right one, then there is a coincidence registered and the counter of the total number of events as well as the B-counter count. Thus, the hidden parameter governs the whole process in such a way that the observed relative frequencies reproduce the probabilities measured in the experiment:

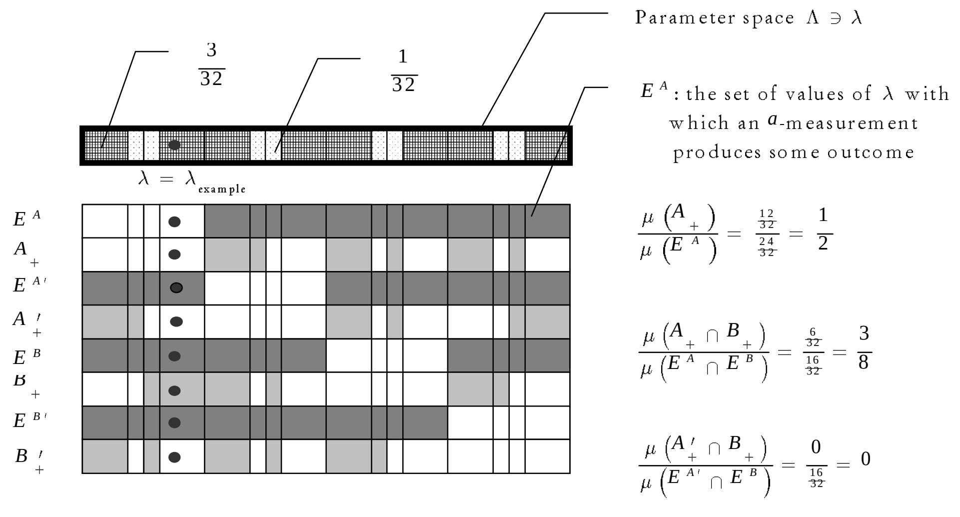

Figure 9 *Parameter $\lambda \in \Lambda$ completely predetermines all future events for all possible combination of the freely chosen measurements.*

$$\frac{\mu(A_+)}{\mu(E^A)} = \frac{\mu(A'_+)}{\mu(E^{A'})} = \frac{\mu(B_+)}{\mu(E^B)} = \frac{\mu(B'_+)}{\mu(E^{B'})} = \frac{\frac{12}{32}}{\frac{24}{32}} = \frac{1}{2}$$

$$\frac{\mu(A_+ \cap B_+)}{\mu(E^A \cap E^B)} = \frac{\mu(A_+ \cap B'_+)}{\mu(E^A \cap E^{B'})} = \frac{\mu(A'_+ \cap B'_+)}{\mu(E^{A'} \cap E^{B'})} = \frac{\frac{6}{32}}{\frac{16}{32}} = \frac{3}{8}$$

$$\frac{\mu(A'_+ \cap B_+)}{\mu(E^{A'} \cap E^B)} = \frac{0}{\frac{16}{32}} = 0$$

Compatibility with the actual EPR experiments

12. As we have seen, the basic idea of the Einstein-Fine interpretation is that some elements of the statistical ensemble of identically prepared quantum systems (characterized by a quantum state W) do not produce outcome at all when one perform the measurement of a quantum observable A. Such systems are called A-*defective* in Fine's terminology. In connection with this basic feature of the model, one can investigate some important characteristics of the above Einstein-Fine model of the EPR experiment, and compare them with the similar characteristics of the actual EPR experiments:

$$R^A = \frac{\text{Number of non-}A\text{-defective systems}}{\text{Total number of systems}}$$

$$R^{A'} = \frac{\text{Number of non-}A'\text{-defective systems}}{\text{Total number of systems}}$$

$$R^B = \frac{\text{Number of non-}B\text{-defective systems}}{\text{Total number of systems}}$$

$$R^{B'} = \frac{\text{Number of non-}B'\text{-defective systems}}{\text{Total number of systems}}$$

$$R^{AB} = \frac{\text{Number of non-}A\text{-defective and non-}B\text{-defective systems}}{\text{Total number of systems}}$$

$$\vdots$$

$$R^{A'B'} = \frac{\text{Number of non-}A'\text{-defective and non-}B'\text{-defective systems}}{\text{Total number of systems}}$$

In case of the above example:

$$\begin{gathered} R^A = R^{A'} = R^B = R^{B'} = 75\% \\ R^{AB} = R^{AB'} = R^{A'B} = R^{A'B'} = 50\% \end{gathered} \tag{6}$$

One can prove (Fine 1982) that the model described in point **11** is maximal in the sense that in any other Einstein-Fine model of the EPR scenario these rates are lower.

A natural question is what are the similar rates in the actual experiments. For if in a real experiment one of these rates was higher than the corresponding one in (6), the Einstein-Fine interpretation would be *experimentally* refuted.

As I mentioned in point **9**, there are principal obstacles to an event ready detection, therefore we cannot have a precise information about the "total number of systems. In one of the best experiments of the last years (Weihs 1998) the *estimated* rates are the following[1]:

$$\begin{aligned} R^{A} = R^{A'} = R^{B} = R^{B'} = 5\% \\ R^{AB} = R^{AB'} = R^{A'B} = R^{A'B'} = 0,25\% \end{aligned} \tag{7}$$

These data are far from victimizing the validity of the Einstein-Fine interpretation!

13. It can be (and probably is) the case that this very low detection/emission rate is caused mostly not by the appearance of the $A(A', B, B')$ -defective particles, but rather by other random effects. There is however enough place left for the systematic mechanism needed for the Einstein-Fine interpretation.

Nevertheless, in order to separate the random effects causing such low detection/emission efficiency, consider a new characteristic of the Einstein-Fine model, which is independent of the rates (7):

$$\begin{aligned} r_{A}^{AB} &= \frac{\text{The number of non-}A\text{-defective and non-}B\text{-defective systems}}{\text{The number of non-}A\text{-defective systems}} \\ r_{A}^{AB'} &= \frac{\text{The number of non-}A\text{-defective and non-}B'\text{-defective systems}}{\text{The number of non-}A\text{-defective systems}} \\ &\vdots \\ r_{B'}^{A'B'} &= \frac{\text{The number of non-}A'\text{-defective and non-}B'\text{-defective systems}}{\text{The number of non-}B'\text{-defective systems}} \end{aligned} \tag{8}$$

In our Einstein-Fine model:

$$r_{A}^{AB} = r_{A}^{AB'} = r_{A'}^{A'B} = r_{A'}^{A'B'} = r_{B}^{AB} = r_{B'}^{AB'} = r_{B}^{A'B} = r_{B'}^{A'B'} = 66,66\%$$

The experiment by Weihs *et al*. (1998) had a particular new feature: At the two wings independent data registration was performed by each observer having his own atomic clock, synchronized only once before each experiment cycle. A time tag was stored for each detected photon in two separate computers at the observer stations and the stored data were analyzed for coincidences long after measurements were finished. Due to this method of data registration, it was possible to count the rates in (8). Again, if any of these rates were higher than 66,66%, the Einstein-Fine interpretation wouldn't be tenable. However the experimental values were only around 5%.

The prism model of an $n \times n$ spin correlation experiment

14. In point **10** I recalled a few objections against Fine's approach, which could be easily answered, since they were rooted in misunderstandings. There is, however, a more serious objection, which is directed against Fine's concrete prism models of the EPR experiment, rather than the general conception of the Einstein-Fine interpretation. However, this objection is more formidable, in fact it sounds quite crushing when one hears it first.

[1] I would like to thank G. Weihs and A. Zeilinger for the private communications about many interesting details of the experiment.

In the EPR experiment we consider only 2×2 different possible directions ($\mathbf{a},\mathbf{a}',\mathbf{b},\mathbf{b}'$). If nature works according to Fine's prism model, then there must exist, in principle, a larger $n\times n$ prism model reproducing all potential 2×2 sub-experiments. It is because nature does not know about how the experiment is designed. On the other hand, Fine (1991) has shown that for a certain class of prism models of the $n\times n$ spin correlation experiment the efficiencies tend to zero, if $n\to\infty$. This contradicts the real experiments. The class of $n\times n$-type prism models for which the above mentioned result is derived is defined via several symmetry conditions. A straightforward generalization of Fine's maximal prism model presented in point **11** belongs to that class. If all physically plausible prism models had to satisfy these symmetry conditions, the problem of zero (or at least very low) efficiencies would mean a serious objection to the Einstein-Fine interpretation.[2]

The rest of the paper is devoted to showing that many of these symmetry conditions can be omitted without loosing the physical plausibility of the model. Then, of course, the question is whether there exists such a prism model for the $n\times n$ spin correlation experiment. I will prove the existence of such models. Moreover, we will see that in these models *the efficiencies do not tend to zero if* $n\to\infty$.

15. The general schema of the prism model of a spin-correlation experiment is the following. At both wings one considers n different possible events:

$$\overbrace{\underbrace{A_1,A_2}_{[A_1]=[A_2]},\underbrace{A_3,A_4}_{[A_3]=[A_4]},\ldots\underbrace{A_{n-1},A_n}_{[A_{n-1}]=[A_n]}}^{\text{left}}\quad\overbrace{\underbrace{B_1,B_2}_{[B_1]=[B_2]},\underbrace{B_3,B_4}_{[B_3]=[B_4]},\ldots\underbrace{B_{n-1},B_n}_{[B_{n-1}]=[B_n]}}^{\text{right}}\tag{9}$$

A_1 denotes the event that the left particle has spin "up" along direction $\mathbf{a}$. A_2 denotes the event that the left particle has spin "down" along direction $\mathbf{a}$. Similarly, A_3 denotes the event that the left particle has spin "up" along direction $\mathbf{a}'$ and A_4 denotes the event that the left particle has spin "down" along direction $\mathbf{a}'$, etc. We have $\frac{n}{2}$ different directions on both sides. We also assume the following logical relationships:

$$\begin{array}{cc}
[A_1]=[A_2]=A_1\vee A_2 & [B_1]=[B_2]=B_1\vee B_2\\
[A_3]=[A_4]=A_3\vee A_4 & [B_3]=[B_4]=B_3\vee B_4\\
\vdots & \vdots\\
[A_{n-1}]=[A_n]=A_{n-1}\vee A_n & [B_{n-1}]=[B_n]=B_{n-1}\vee B_n\\
A_1\wedge A_2=0 & B_1\wedge B_2=0\\
A_3\wedge A_4=0 & B_3\wedge B_4=0\\
\vdots & \vdots\\
A_{n-1}\wedge A_n=0 & B_{n-1}\wedge B_n=0
\end{array}\tag{10}$$

that is, $[A_1]$ (which is equal to $[A_2]$) denotes the event that the left particle is predetermined to produce any outcome if the $\mathbf{a}$ direction is measured. The quantum probabilities are reproduced in the following way:

[2] The low (zero) efficiency problem appearing in particular prism models is frequently used as the final argument against the Einstein-Fine interpretation. Cf. Sharp and Shanks 1985; Maudlin 1994, Section 6.

$$
\begin{aligned}
tr\left(WP_{A_i}\right) &= q_i = \frac{p\left(A_i\right)}{p\left(\left[A_i\right]\right)} \\
tr\left(WP_{B_i}\right) &= q_i' = \frac{p\left(B_i\right)}{p\left(\left[B_i\right]\right)} \\
tr\left(WP_{A_i}P_{B_j}\right) &= q_{ij} = \frac{p\left(A_i \wedge B_j\right)}{p\left(\left[A_i\right] \wedge \left[B_j\right]\right)}
\end{aligned}
\tag{11}
$$

The quantum probabilities $q_1, q_2, \ldots q_1', q_2', \ldots q_{ij}, \ldots$ are the only fix numbers in the model.

The question I want to investigate is whether there exists such a prism model with reasonable efficiencies $p\left(\left[A_i\right]\right)$ and $p\left(\left[A_i\right] \wedge \left[B_j\right]\right)$ for all $n \times n$ experiments. I mean by "reasonable" that the efficiencies do not tend to zero when $n \to \infty$.

16. Fine's prism model of the 2×2 EPR experiment satisfies the following symmetry conditions, too:

$$
p\left(\left[A_i\right]\right) = \omega \qquad \text{for all } 0 \le i \le n \tag{12}
$$

$$
p\left(\left[A_i\right] \wedge \left[B_j\right]\right) = \sigma \qquad \text{for all } 0 \le i, j \le n \tag{13}
$$

where ω and σ are some uniform efficiencies for all directions on both sides. We will require the uniformity (12), since it is completely justified by the symmetry of the experimental setup. We will also require (13), although it is something more than what follows from the symmetry of the setup. Contrary to Fine (1991), conditions (12)-(13) are the only symmetries we prescribe for a prism model. In point **20** I shall explain why.

Existence of reasonable prism models

17. Obviously, a prism model can exist if and only if there exists a suitable Kolmogorov probability space which is capable to accommodate events (9) together with relations (10) and the probability measure of which satisfies the requirements (11). In addition, we require the symmetry conditions (12) and (13).

It follows from the Pitowsky theorem[3] that such a Kolmogorov probability space can exists if and only if the following correlation vector is classical, that is, it is contained in the corresponding classical correlation polytope:

[3] In this section I will use the notations and several theorems from Pitowsky 1989, Chapters 2 and 3.

$$p = \begin{pmatrix} p(A_1) \\ \vdots \\ p(A_n) \\ p(B_1) \\ \vdots \\ p(B_n) \\ p([A_2]) \\ p([A_4]) \\ \vdots \\ p([B_n]) \\ p(A_1 \wedge B_1) \\ \vdots \\ p(A_i \wedge B_j) \\ \vdots \\ p(A_n \wedge B_n) \\ p([A_2] \wedge [B_2]) \\ p([A_2] \wedge [B_4]) \\ \vdots \\ p([A_{2k}] \wedge [B_{2l}]) \\ \vdots \\ p([A_n] \wedge [B_n]) \\ p(A_1 \wedge [A_1]) \\ \vdots \\ p(A_i \wedge [A_i]) \\ \vdots \\ p(B_j \wedge [B_j]) \\ \vdots \\ p(B_n \wedge [B_n]) \\ p(A_1 \wedge A_2) \\ p(A_3 \wedge A_4) \\ \vdots \\ p(A_{n-1} \wedge A_n) \\ p(B_1 \wedge B_2) \\ p(B_3 \wedge B_4) \\ \vdots \\ p(B_{n-1} \wedge B_n) \end{pmatrix} = \begin{pmatrix} q(A_1)\, p([A_1]) \\ \vdots \\ q(A_n)\, p([A_n]) \\ q(B_1)\, p([B_1]) \\ \vdots \\ q(B_n)\, p([B_n]) \\ p([A_2]) \\ p([A_4]) \\ \vdots \\ p([B_n]) \\ q(A_1 \wedge B_1)\, p([A_1] \wedge [B_1]) \\ \vdots \\ q(A_i \wedge B_j)\, p([A_i] \wedge [B_j]) \\ \vdots \\ q(A_n \wedge B_n)\, p([A_n] \wedge [B_n]) \\ p([A_2] \wedge [B_2]) \\ p([A_2] \wedge [B_4]) \\ \vdots \\ p([A_{2k}] \wedge [B_{2l}]) \\ \vdots \\ p([A_n] \wedge [B_n]) \\ q(A_1)\, p([A_1]) \\ \vdots \\ q(A_i)\, p([A_i]) \\ \vdots \\ q(B_j)\, p([B_j]) \\ \vdots \\ q(B_n)\, p([B_n]) \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} = \begin{pmatrix} q(A_1)\omega \\ \vdots \\ q(A_n)\omega \\ q(B_1)\omega \\ \vdots \\ q(B_n)\omega \\ \omega \\ \omega \\ \vdots \\ \omega \\ q(A_1 \wedge B_1)\sigma \\ \vdots \\ q(A_i \wedge B_j)\sigma \\ \vdots \\ q(A_n \wedge B_n)\sigma \\ \sigma \\ \sigma \\ \vdots \\ \sigma \\ \vdots \\ \sigma \\ q(A_1)\omega \\ \vdots \\ q(A_i)\omega \\ \vdots \\ q(B_j)\omega \\ \vdots \\ q(B_n)\omega \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \in C(3n, S) \qquad (14)$$

There is, of course, a trivial and meaningless solution when all efficiencies are zero. We also know that in case of $\omega = \sigma = 1$ the correlation vector in (14) is, in general, non-classical:

$$
q = \begin{pmatrix} q(A_1) \\ \vdots \\ q(A_i) \\ \vdots \\ q(B_j) \\ \vdots \\ q(B_n) \\ 1 \\ \vdots \\ 1 \\ q(A_1 \wedge B_1) \\ \vdots \\ q(A_i \wedge B_j) \\ \vdots \\ q(A_n \wedge B_n) \\ 1 \\ \vdots \\ 1 \\ q(A_1) \\ \vdots \\ q(A_i) \\ \vdots \\ q(B_j) \\ \vdots \\ q(B_n) \\ 0 \\ \vdots \\ 0 \end{pmatrix} \notin C(3n,S) \qquad (15)
$$

18. The main new result of this paper can be formulated in the following theorem:

Theorem *For arbitrary quantum correlation vector q in (15) there exist uniform efficiencies* $0<\omega,\sigma<1$ *such that condition (14) is satisfied.*

Proof Since $q \in Q(3n,S)$, it can be decomposed as a convex linear combination of the vertices of the quantum correlation polytope:

$$
q = \sum_{\varepsilon\in\{0,1\}^{3n}} \lambda_\varepsilon u^\varepsilon + \sum_\alpha \tilde{\lambda}_\alpha u^\alpha \qquad \left(\sum_\varepsilon \lambda_\varepsilon + \sum_\alpha \tilde{\lambda}_\alpha = 1 \text{ and } \lambda_\varepsilon, \tilde{\lambda}_\alpha \geq 0\right)
$$

Each non-classical vertex u^α can be decomposed as the sum of the following two classical vertices:

$$
\tilde{u}^{\alpha} = \begin{pmatrix} \tilde{u}^{\alpha}(A_1) \\ \vdots \\ \tilde{u}^{\alpha}(A_i) \\ \vdots \\ \tilde{u}^{\alpha}(B_j) \\ \vdots \\ \tilde{u}^{\alpha}(B_n) \\ 1 \\ \vdots \\ 1 \\ \tilde{u}^{\alpha}(A_1 \wedge B_1) \\ \vdots \\ \tilde{u}^{\alpha}(A_i \wedge B_j) \\ \vdots \\ \tilde{u}^{\alpha}(A_n \wedge B_n) \\ 1 \\ \vdots \\ 1 \\ \tilde{u}^{\alpha}(A_1) \\ \vdots \\ \tilde{u}^{\alpha}(A_i) \\ \vdots \\ \tilde{u}^{\alpha}(B_j) \\ \vdots \\ \tilde{u}^{\alpha}(B_n) \\ 0 \\ \vdots \\ 0 \end{pmatrix} = \underbrace{\begin{pmatrix} \tilde{u}^{\alpha}(A_1) \\ \vdots \\ \tilde{u}^{\alpha}(A_i) \\ \vdots \\ \tilde{u}^{\alpha}(B_j)\delta_j \\ \vdots \\ \tilde{u}^{\alpha}(B_n)\delta_n \\ 1 \\ \vdots \\ 1 \\ \tilde{u}^{\alpha}(A_1 \wedge B_1) \\ \vdots \\ \tilde{u}^{\alpha}(A_i \wedge B_j) \\ \vdots \\ \tilde{u}^{\alpha}(A_n \wedge B_n) \\ 1 \\ \vdots \\ 1 \\ \tilde{u}^{\alpha}(A_1) \\ \vdots \\ \tilde{u}^{\alpha}(A_i) \\ \vdots \\ \tilde{u}^{\alpha}(B_j) \\ \vdots \\ \tilde{u}^{\alpha}(B_n) \\ 0 \\ \vdots \\ 0 \end{pmatrix}}_{\overline{u}^{\alpha}} + \underbrace{\begin{pmatrix} 0 \\ \vdots \\ 0 \\ \vdots \\ \tilde{u}^{\alpha}(B_j)(1-\delta_j) \\ \vdots \\ \tilde{u}^{\alpha}(B_n)(1-\delta_n) \\ 0 \\ \vdots \\ 0 \\ 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}}_{\overline{\overline{u}}^{\alpha}}
$$

where

$$
\delta_j = \begin{cases} 1 & \text{if } (\exists i)\left[\tilde{u}\left(A_i \wedge B_j\right) = 1\right] \\ 0 & \text{otherwise} \end{cases}
$$

The non-classical correlation vector q can be written as:

$$
q = \sum_{\varepsilon \in \{0,1\}^{2n}} \lambda_{\varepsilon} u^{\varepsilon} + \sum_{\alpha} \tilde{\lambda}_{\alpha} \left(\overline{u}^{\alpha} + \overline{\overline{u}}^{\alpha}\right) = \sum_{\varepsilon \in \{0,1\}^{2n}} \lambda_{\varepsilon} u^{\varepsilon} + \sum_{\alpha} \tilde{\lambda}_{\alpha} \overline{u}^{\alpha} + \sum_{\alpha} \tilde{\lambda}_{\alpha} \overline{\overline{u}}^{\alpha}
$$

That is, q is a weighted sum of classical vertices with weights the sum of which is larger than 1. On the other hand, we know that

$$
\sum_{\varepsilon \in \{0,1\}^{2n}} \lambda_{\varepsilon} + 2\sum_{\alpha} \tilde{\lambda}_{\alpha} \le 2 \tag{16}
$$

Consider the following correlation vector:

$$p_I = xq = \sum_{\varepsilon\in\{0,1\}^{2n}} x\lambda_\varepsilon u^\varepsilon + \sum_\alpha x\tilde{\lambda}_\alpha \bar{u}^\alpha + \sum_\alpha x\tilde{\lambda}_\alpha \bar{\bar{u}}^\alpha$$

It follows from (16) that $p_I \in C(3n,S)$ if $x \le \frac{1}{2}$. Consequently, we can write that

$$p_I = xq = \sum_{\varepsilon\in\{0,1\}^{3n}} \mu_\varepsilon u^\varepsilon \qquad \left(\sum_\varepsilon \mu_\varepsilon = 1 \text{ and } \mu_\varepsilon \ge 0\right) \tag{17}$$

Let us introduce the following projectors in $R(3n,S)$:

$$p = \begin{pmatrix} p(A_1) \\ \vdots \\ p(A_i) \\ \vdots \\ p(B_j) \\ \vdots \\ p(B_n) \\ p([A_2]) \\ \vdots \\ p([A_{2k}]) \\ \vdots \\ p([B_{2l}]) \\ \vdots \\ p([B_n]) \\ p(A_1 \wedge B_1) \\ \vdots \\ q(A_i \wedge B_j) \\ \vdots \\ q(A_n \wedge B_n) \\ p([A_1]\wedge[B_1]) \\ \vdots \\ p([A_{2k}]\wedge[B_{2l}]) \\ \vdots \\ p([A_n]\wedge[B_n]) \\ p(A_1 \wedge [A_1]) \\ \vdots \\ p(A_i \wedge [A_i]) \\ \vdots \\ p(B_j \wedge [B_j]) \\ \vdots \\ p(B_n \wedge [B_n]) \\ 0 \\ \vdots \\ 0 \end{pmatrix} \quad \mapsto \quad \Pi^A p = \begin{pmatrix} p(A_1) \\ \vdots \\ p(A_i) \\ \vdots \\ 0 \\ \vdots \\ 0 \\ p([A_2]) \\ \vdots \\ p([A_{2k}]) \\ \vdots \\ 0 \\ \vdots \\ 0 \\ 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \\ 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \\ p(A_1 \wedge [A_1]) \\ \vdots \\ p(A_i \wedge [A_i]) \\ \vdots \\ 0 \\ \vdots \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

and

$$p = \begin{pmatrix} p(A_1) \\ \vdots \\ p(A_i) \\ \vdots \\ p(B_j) \\ \vdots \\ p(B_n) \\ p([A_2]) \\ \vdots \\ p([A_{2k}]) \\ \vdots \\ p([B_{2l}]) \\ \vdots \\ p([B_n]) \\ p(A_1 \wedge B_1) \\ \vdots \\ q(A_i \wedge B_j) \\ \vdots \\ q(A_n \wedge B_n) \\ p([A_1] \wedge [B_1]) \\ \vdots \\ p([A_{2k}] \wedge [B_{2l}]) \\ \vdots \\ p([A_n] \wedge [B_n]) \\ p(A_1 \wedge [A_1]) \\ \vdots \\ p(A_i \wedge [A_i]) \\ \vdots \\ p(B_j \wedge [B_j]) \\ \vdots \\ p(B_n \wedge [B_n]) \\ 0 \\ \vdots \\ 0 \end{pmatrix} \mapsto \Pi^B p = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ \vdots \\ p(B_j) \\ \vdots \\ p(B_n) \\ 0 \\ \vdots \\ 0 \\ \vdots \\ p([B_{2l}]) \\ \vdots \\ p([B_n]) \\ 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \\ 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \\ 0 \\ \vdots \\ 0 \\ \vdots \\ p(B_j \wedge [B_j]) \\ \vdots \\ p(B_n \wedge [B_n]) \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

Consider the following correlation vector:

$$p_{II} = \sum_{\varepsilon \in \{0,1\}^{3n}} y \mu_\varepsilon u^\varepsilon + \sum_{\varepsilon \in \{0,1\}^{3n}} \frac{1-y}{2} \mu_\varepsilon \Pi^A u^\varepsilon + \sum_{\varepsilon \in \{0,1\}^{3n}} \frac{1-y}{2} \mu_\varepsilon \Pi^B u^\varepsilon$$

$$= \begin{pmatrix} q(A_1)\left(xy - x\frac{1-y}{2}\right) \\ \vdots \\ q(A_i)\left(xy - x\frac{1-y}{2}\right) \\ \vdots \\ q(B_j)\left(xy - x\frac{1-y}{2}\right) \\ \vdots \\ q(B_n)\left(xy - x\frac{1-y}{2}\right) \\ \left(xy - x\frac{1-y}{2}\right) \\ \vdots \\ \left(xy - x\frac{1-y}{2}\right) \\ q(A_1 \wedge B_1)xy \\ \vdots \\ q(A_i \wedge B_j)xy \\ \vdots \\ q(A_n \wedge B_n)xy \\ xy \\ \vdots \\ xy \\ q(A_1)\left(xy - x\frac{1-y}{2}\right) \\ \vdots \\ q(A_i)\left(xy - x\frac{1-y}{2}\right) \\ \vdots \\ q(B_j)\left(xy - x\frac{1-y}{2}\right) \\ \vdots \\ q(B_n)\left(xy - x\frac{1-y}{2}\right) \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

where the μ_ε's are taken from (17). Taking into account that for all $\varepsilon \in \{0,1\}^{3n}$, $\Pi^A u^\varepsilon$ and $\Pi^B u^\varepsilon$ are classical vertices, too, p_{II} is a weighted sum of classical vertices. Moreover,

$$\sum_{\varepsilon\in\{0,1\}^{3n}} y\mu_\varepsilon + \sum_{\varepsilon\in\{0,1\}^{3n}} \frac{1-y}{2}\mu_\varepsilon + \sum_{\varepsilon\in\{0,1\}^{3n}} \frac{1-y}{2}\mu_\varepsilon = 1$$

Therefore, $p_{II} \in C(3n,S)$ for all x and y satisfying that

$$\begin{aligned} x &\le \frac{1}{2} \\ y &\le 1 \end{aligned} \tag{18}$$

Now, let the efficiencies be $\omega = \left(xy + x\frac{1-y}{2} \right)$ and $\sigma = xy$.

■

19. In order to clarify what kinds of efficiencies are allowed, express the conditions (18) in terms of ω and σ:

$$\begin{aligned} \sigma &\le \omega \\ 4\omega - 2\sigma &\le 1 \end{aligned} \tag{19}$$

It follows from (19) that the maximal efficiency case is when $\sigma = \omega = \frac{1}{2}$. It can be also interesting when the events $[A_i]$ on the left hand side are statistically independent of the events $[B_j]$ on the right hand side, that is,

$$\sigma = \omega^2 \tag{20}$$

In this case we have the following upper bound for the efficiency:

$$\omega \le \frac{2-\sqrt{2}}{2} \cong 0.29$$

An important feature of the condition (19) is that it is independent of n. Consequently, *it is not true that the efficiency must tend to zero if* $n \to \infty$. It is no less remarkable that (19) is a sufficient but *not necessary* condition of the existence of a suitable prism model. In other words, the above-derived results do not exclude the existence of models with higher efficiencies. Although, as we have seen in points **12** and **13**, the above limits to the efficiencies, the sufficiency of which we proved, are much higher than the detection/emission efficiencies in the recent spin-correlation experiments.

20. The above result prima facie contradicts Fine's (1991) similar analysis. From formal, mathematical point of view, however, there appears no contradiction. Since Fine investigated a particular class of prism models satisfying very strong symmetry conditions, called *Exchangeability*, *Indifference*, and *Strong Symmetry.* In my view, these conditions are completely unjustified, since they do not follow from the physical symmetries of the experimental setup. In fact, these conditions were derived from the following one single assumption:

$$p\left([A_i] \,\|\, \text{hidden parameter}\right) = p\left([B_i] \,\|\, \text{hidden parameter}\right) \qquad \text{for all } i = 1,2,\ldots n \tag{21}$$

Assumption (21) is not only implausible from physical point of view, but it is absolutely untenable, in general: In case of a deterministic hidden variable model, like the prism models we constructed, it contradicts Fine's another assumption, namely, the independence (20). Because in deterministic case (21) means that in each state of affairs (at each value of hidden parameter) events $[A_i]$ and $[B_i]$ simultaneously do or do not occur. Consequently they cannot be statistically independent.

Conclusions

We proved that, contrary to what has been often claimed in the literature, there exist prism models for the $n \times n$ spin correlation experiment, such that the physically plausible symmetry conditions (12)-(13) are satisfied. They may also satisfy the independence condition (20). At the same time, the efficiencies do not tend to zero if $n \to \infty$. By proving this result we removed the only serious obstacle to the Einstein-Fine solution of the EPR-Bell paradox. The Einstein-Fine interpretation resolves the contradiction between the violation of Bell-type inequalities and the assumption that the EPR experiment can be accommodated in a relativistic and deterministic universe.

I claimed that the real EPR experiments as well as many others, perhaps all, quantum measurements have such a logical scheme, which admits the Einstein-Fine interpretation. And it is not a "metaphysical" claim; we derived several conditions, the violation of which would imply the refutation of the Einstein-Fine interpretation of quantum mechanics. However, it turned out that there is no experimental indication of the violation of these conditions in the actual EPR experiments.

With this paper I wanted to contribute to the rediscovery of Arthur Fine's first rediscovery of Einstein's realistic statistical interpretation of quantum mechanics. The Einstein-Fine interpretation seems to be the only tenable interpretation of quantum mechanics. It is free from those difficulties and contradictions, which the other rival interpretations suffer from: a) As a statistical interpretation, it is devoid of the measurement paradox and the likes. b) It is free from the Kochen-Specker-type contradictions, since it is a realistic interpretation admitting that the individual quantum systems have pre-settled intrinsic properties, prior to, independent of and revealed by the measurements. c) All probabilities can be interpreted as relative frequencies in a well-defined ordinary statistical ensemble. The "quantum probabilities", too, obtain a meaningful explanation inside of the classical Kolmogorov theory of probability. d) The Einstein-Fine interpretation is local in the sense that it does not contradict to a relativistic and deterministic picture I described in point **6**. e) It admits, without contradiction, a local deterministic hidden variable theory.

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